

**Complex Rational Expressions**

**Complex rational expression** – a rational expression whose numerator, denominator or both contain one or more rational expressions.

**Simplifying a complex fraction (Method 1: Multiplying by 1)**

Multiply the numerator and denominator of the complex fraction by the LCD of the fractions in both numerator and denominator

$$\frac{\frac{1}{a^3b} + \frac{1}{b}}{\frac{1}{a^2b^2} - \frac{1}{b^2}} = \frac{\left(\frac{1}{a^3b} + \frac{1}{b}\right) \cdot a^3b^2}{\left(\frac{1}{a^2b^2} - \frac{1}{b^2}\right) \cdot a^3b^2} = \frac{\left(\frac{1}{a^3b} \cdot a^3b^2 + \frac{1}{b} \cdot a^3b^2\right)}{\left(\frac{1}{a^2b^2} \cdot a^3b^2 - \frac{1}{b^2} \cdot a^3b^2\right)}$$

$$= \frac{b + a^3b}{a - a^3} = \frac{b(1 + a^3)}{a(1 - a^2)} = \frac{b(1 + a)(1 + a + a^2)}{a(1 - a)(1 + a)} = \frac{b(1 + a + a^2)}{a(1 - a)}$$

OR

**Simplifying a complex fraction (Method 2: Dividing Two Rational Expressions)**

- A. Simplify the numerator and the denominator of the complex fraction so that each is a single fraction
- B. Perform the indicated division by multiplying the numerator of the complex fraction by the reciprocal of the denominator.
- C. Simplify if possible

OR

**Examples:**

$$\frac{\frac{x-3}{x+4}}{\frac{x+6}{x-1}}$$

$$\frac{\frac{a^2 - b^2}{ab}}{\frac{a-b}{b}}$$

$$\frac{\frac{a^{-1} + b^{-1}}{a^2 - b^2}}{ab}$$

$$\frac{\frac{1}{y} + 2}{\frac{1}{y} - 3}$$

$$\frac{\frac{x}{x^2 + 5x - 6} + \frac{6}{x^2 + 5x - 6}}{\frac{x}{x^2 - 5x + 4} + \frac{6}{x^2 - 5x + 4}}$$

$$\frac{a(a+3)^{-1} - 2(a-1)^{-1}}{a(a+3)^{-1} - (a-1)^{-1}}$$